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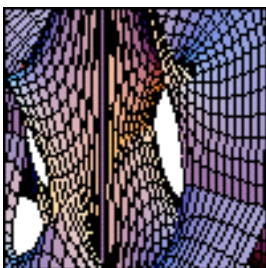
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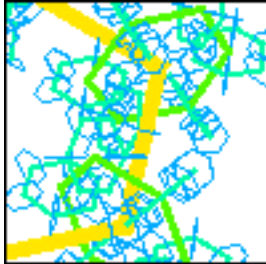
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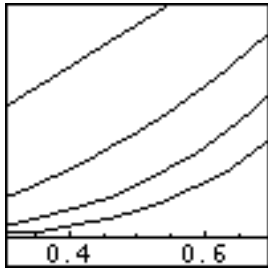
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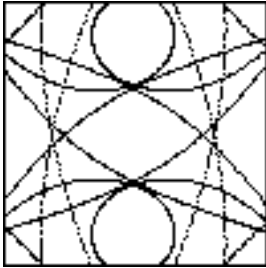
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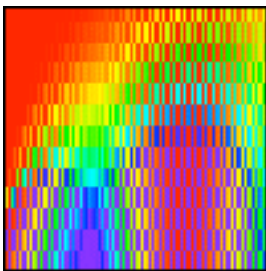
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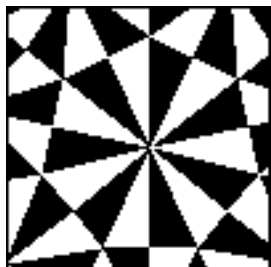
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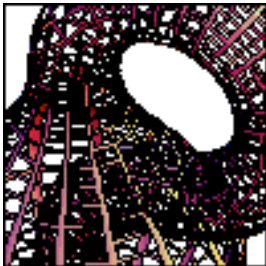
▷ ▷ Exercises

Game of Life ■ Langton's Ant ■ Brillouin Zones ■ Maxwell–Helmholtz Color Triangle ■ Conformal Maps ■ Cornet Isogons ■ Jarník Polygons ■ Light Ray Reflections in a Water Drop ■ Warped Patterns ■ Moiré Patterns ■ Triptych Fractal ■ Multiple Reflected Pentagons ■ Random Lissajous Figures ■ Walsh Function ■ Sorting Game ■ Ball Moves ■ Rectangle Packings ■ Smoothed L-Systems ■ Polygonal Billiards ■ Random Walk on a Sierpinski Fractal ■ Voronoi Tessellations ■ Lévy Flights ■ Random Supersymmetric Potential ■ Common Plotting Problems ■ Nomogram for Quadratic Equation ■ Clusters on Square Grids ■ Aperiodic Triangle Tilings

▷ ▷ Solutions

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▷ ▷ References



CHAPTER 2

Three–Dimensional Graphics

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▷ ▷ Overview

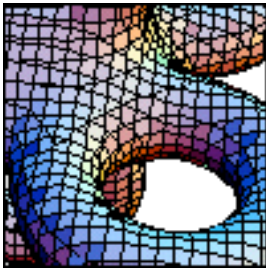
▷ ▷ Exercises

3D Surface Sampler ■ Warped, Twisted, and Interlocked Tori ■ Dodecahedra Iteratively Reflected on its Faces ■ Snail ■ Trinomial Theorem Visualization ■ Ball Blending Method ■ Loop Subdivision ■ $\sqrt{3}$ -Subdivision Algorithm ■ Averaging Closed Curves ■ Projective Plane Model ■ Counting Surfaces for a Given Genus ■ Lattice Pyramids ■ Fractal Mountains ■ Random Walk on a Sphere ■ Projecting onto Polyhedra ■ Alexander's Horned Sphere ■ Polyhedral Caustic ■ Sliced Möbius Strip ■ Perspective Modeling ■ Displaying Hidden Edges ■ Generating Platonic Solid Clusters ■ A 4D Platonic Solid—The 120-Cell ■ Folding a Dodecahedron ■ Continuously Changing Polyhedra ■ Inscribing Five Cubes in a Dodecahedron ■ Interwoven Bands around a Dodecahedron ■ Knot Made from Knots ■ Knot with Escher Tiling ■ Gear Chain Animation ■ 3D Peano Polygon ■ Tetraview Riemann Surface Animation ■ Riemann Surface of Kepler Equation ■ Sierpinski Plant

▷ ▷ Solutions

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▷ ▷ **References**



CHAPTER 3

Contour and Density Plots

3.0 Remarks

3.1 Contour Plots

Contour Graphics ■ Converting Contour Graphics ■ Options of Contour Graphics ■ Cassini Curve ■ Various Sample Contour Plots ■ Functions Varying Strongly ■ Homogeneous Contour Line Density ■ Coloring Contour Plots ■ Contour Graphics in Nonrectangular Domains ■ Speckles and Scarlets from Superimposing 2D Waves ■ Smoothing Contour Lines ■ Superimposed 2D Waves in Symmetric Directions ■ Comparing Options and Option Settings of Plotting Functions ■ Algebraic Description of Polygons ■ Blaschke Products ■ Charged Goffinet Dragon ■ Square Well-Scattering Amplitude

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▷ ▷ **Overview**

▷ ▷ **Exercises**

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▷ ▷ **Solutions**

Visualizing Saddle Points ■ Outer Products ■ Repeatedly Mirrored Matrix ■ Halley Map ■ Generating Random Functions ■ Weierstrass \wp Function Based Fractal ■ Contour Plots in Non-Cartesian Coordinate Systems ■ Spheres with Handles ■ Cmutov Surfaces ■ Random Surfaces with Dodecahedral Symmetry ■ Polynomials over the Riemann Sphere ■ Random Radial-Azimuthal Transition ■ Contour Lines in 3D Plots ■ Lines on Polygons ■ Slicing Surfaces ■ Euler–Poincaré Formula ■ Mapping Disks to Polygons ■ Statistics of n -gons in 3D Contour Plots

▷ ▷ **References**

N U M E R I C S



CHAPTER 1 *Numerical Computations*

1.0 Remarks

Summing Machine Numbers ■ Klein's Modular Function and Chazy Equation ■ Discretizing the Rössler System ■ Modeling the Ludwig–Soret Effect

1.1 Approximate Numbers

1.1.0 Remarks

1.1.1 Numbers with an Arbitrary Number of Digits

Machine Arithmetic versus High-Precision Arithmetic ■ Modified Logistic Map ■ Numerical Calculation of Weierstrass Functions ■ High-Precision Arithmetic System Parameters ■ Fixed-Precision Arithmetic ■ Random Fibonacci Recursion ■ Smart Numericalization ■ Precision and Accuracy of Real Numbers ■ Precision and Accuracy of Complex Numbers ■ Precision Loss and Gain in Calculations ■ Error Propagation in Numerical Calculations ■ Principles of Significance Arithmetic ■ Error Propagation for Multivariate Functions ■ Collapsing Numeric Expressions ■ Setting Precision and Accuracy of Numbers ■ Guard Digits in High-Precision Numbers ■ The Bits of a Number ■ Sum-Based Methods of Calculating π ■ Comparing High-Precision Numbers ■ Automatic Switching to High-Precision Arithmetic

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Finite Resistor Network ■ Exact versus Approximate Solutions ■ Avoiding Numericalization of Indices ■ Calculating Resistances Through Eigenvalues ■ Tagaki Function ■ Numerical Solution of a Functional Equation ■ Fixed-Precision Arithmetic in Linear Algebra ■ Modular Equation for Klein's Modular Function ■ Null Spaces of Linear Systems ■ Bound State in a Waveguide Crossing ■ Sparse Matrices ■ Square Network with Random Resistance Values ■ Anderson Model

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1.6 Numerical Functions and Their Options

Common Options of Numerical Functions ■ Precision To Be Used in Calculations ■ Machine Precision versus High-Precision ■ Precision Goal for a Numerical Calculation ■ Accuracy Goal for a Numerical Calculation ■ Accuracy Goals for Independent and Dependent Variables ■ Monitoring Numerical Calculations ■ Evaluation Order in Numerical Function ■ Avoiding the Evaluation of the First Argument ■ Using Vector-Valued Variables ■ Dummy Variable-Free Function Calls

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1.9 Solution of Equations

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Boundary and Initial Value Problems ■ Interpolating Functions as Solutions ■ Differential-Algebraic Equations ■ Pendulum ODE ■ Anharmonic Oscillator with Random Forcing ■ Squatting on a Swing ■ Newton Vector Field ■ Spiral Waves ■ 4D Chaotic Attractor ■ Energy Bands in a Random Complex Potential ■ Stiff and Nonstiff Systems ■ Precision Control ■ Nonlinear Differential Equation with Isochronous Solutions ■ Buchstab Function ■ Higher Order ODEs ■ Ablowitz–Ladik Chain ■ Particle Motion in a Wave Field ■ Chazy Equation ■ Boundaries of Analyticity ■ Generalized Airy Functions ■ Monitoring Numerical Differential Equation Solving ■ Stepsize Control ■ Coupled Pendulums ■ Restricting the Solutions ■ Stopping the Solution Process ■ Calculating and Visualizing Pursuits ■ Finding the Initial Slope for the Thomas–Fermi Equation ■ Forced Coupled Oscillators ■ Chaotic Scattering on a Four-Hill Potential ■ Events in Differential Equation Solving ■ Vector and Matrix Differential Equations ■ Method Option Choices ■ Integrated Brownian Motion ■ Modified Lorenz System ■ Calculating Contour Curves Through Differential Equations ■ Geodesics on a Triple-Periodic Surface ■ Using Homotopies to Solve Polynomial Systems ■ Modeling Newton's Cradle ■ Trajectories in Central Force Fields ■ Three-Body Scattering ■ Interacting Vortices ■ Periodic Orbits of the Restricted Three-Body Problem ■ Combining Numerical Functions ■ Periodic Orbits of the Lorenz System ■ Bohm's Quantum Trajectories ■ Continuous Time Random Walks on Graphs ■ Sparse Arrays in Differential Equations

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Algebraic Functions as Bivariate Polynomials ■ Faithful Riemann Surfaces ■ Implicit Parametrizations ■ Branch Cuts and Branch Points ■ Discriminant ■ First Order ODEs for Algebraic Functions ■ Sheets of Riemann Surfaces ■ Samples of Riemann Surfaces

▷ ▷ **Overview**

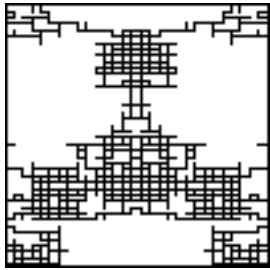
▷ ▷ **Exercises**

Logistic Map ■ Randomly Perturbed Iterative Maps ■ Functions with Boundaries of Analyticity ■ q -Trigonometric Functions ■ Franel Identity ■ Bloch Oscillations ■ Courtright Trick ■ Hannay Angle ■ Harmonic Nonlinear Oscillators ■ Orbits Interpolating Between Harmonic Oscillator and Kepler Potential ■ Shooting Method for Quartic Oscillator ■ Eigenvalues of Symmetric Tridiagonal Matrices ■ Optimized Harmonic Oscillator Expansion ■ Diagonalization in the Schwinger Representation ■ Möbius Potential ■ Bound States in the Continuum ■ Wynn's Epsilon Algorithm ■ Aitken Transformation ■ Numerical Regularization ■ Scherk's Fifth Surface ■ Clebsch Surface ■ Smoothed Dodecahedron Wireframe ■ Standard Map ■ Stochastic Webs ■ Forced Logistic Map ■ Web Map ■ Strange Attractors ■ Hénon Map ■ Triangle Map Basins ■ Trajectories in 2D Periodic Potentials ■ Egg Crate Potential ■ Pearcey Integral ■ Charged Square and Hexagonal Grids ■ Ruler on Two Fingers ■ Branched Flows in Random Potentials ■ Maxwell Line ■ Iterated Secant Method Steps ■ Unit Sphere Inside a Unit Cube ■ Ising-Model Integral ■ Random Binary Trees ■ Random Matrices ■ Iterated Polynomial Roots ■ Weierstrass Root Finding Method ■ Animation of Newton Basins ■ Lagrange Remainder of Taylor Series ■ Nodal Lines ■ Bloch Equations ■ Branch Cuts of Hyperelliptic Curves ■ Strange 4D Attractors ■ Billiard with Gravity ■ Schwarz–Riemann Minimal Surface ■ Jorge–Meeks Trinoid ■ Random Minimal Surfaces ■ Precision Modeling ■ Infinite Resistor Networks ■ Auto-Compiling Functions ■ Card Game Modeling ■ Charges With Cubical Symmetry on a Sphere ■ Tricky Questions ■ Very High-Precision Quartic Oscillator Ground State ■ 1D Ideal Gas ■ Odlyzko–Stanley Sequences ■ Tangent Products ■ Thompson's Lamp ■ Parking Cars ■ Seceder Model ■ Avoided Patterns in Permutations ■ Cut Sequences ■ Exchange Shuffles ■ Frog Model ■ Second Arcsine Law ■ Average Brownian Excursion Shape ■ ABC-System ■ Vortices on a Sphere ■ Oscillations of a Triangular Spring Network ■ Lorenz System ■ Fourier Differentiation ■ Fourier Coefficients of Klein's Function ■ Singular Moduli ■ Curve Thickening ■ Random Textures ■ Random Cluster Growth ■ First Digit Frequencies in Mandelbrot Set Calculation ■ Interesting Jerk Functions ■ Initial Value Problems for the Schrödinger Equation ■ Initial Value Problems for 1D, 2D, and 3D Wave Equation ■ Continued Inverse Square Root Expansion ■ Lüroth Expansion ■ Lehner Expansion ■ Brjuno Function ■ Sum of Continued Fraction Convergents Errors ■ Average Scaled Continued Fraction Errors ■ Bolyai Expansion ■ Symmetric Continued Fraction Expansion

▷ ▷ **Solutions**

Solving Polynomials Using Differential Equations ■ Stabilizing Chaotic Sequences ■ Oscillator Clustering ■ Transfer Matrices ■ Avoided Eigenvalue Crossings ■ Hellmann–Feynman Theorem ■ Scherk Surface Along a Knot ■ Time-Evolution of a Localized Density Under a Discrete Map ■ Automatic Selection of “Interesting” Graphic ■ Gradient Fields ■ Static and Kinematic Friction ■ Smoothing Functions ■ Eigenvalues of Random Binary Trees ■ Basins of Attraction Fractal Iterations ■ Calculating Contour Lines Through Differential Equations ■ Manipulating Downvalues at Runtime ■ Path of Steepest Descent ■ Fourier Series Arc Length ■ Poincaré Sections ■ Random Stirring ■ Heegner Numbers ■ Quantum Random Walk ■ Quantum Carpet ■ Coherent State in a Quantum Well

▷ ▷ **References**



CHAPTER 2

Computations with Exact Numbers

2.0 Remarks

Using Approximate Numerics in Exact Calculations ■ Integer Part Map ■ Misleading Patterns ■ Primes in Quadratic Polynomials

2.1 Divisors and Multiples

Factoring Integers ■ Number of Prime Factors ■ Divisors ■ Sum of Squares ■ Derivative of an Integer ■ mod Function ■ Rotate and Mod ■ n th Digit of a Proper Fraction ■ Schönberg's Peano Curve ■ Greatest Common Divisors and Least Common Multiples ■ Euclidean Algorithm ■ Classical and Generalized Maurer Roses ■ de Bruijn Medallions and Friezes

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Prime Numbers ■ Prime Number Spiral ■ Prime Counting Function ■ Euler's Totient Function ■ Absolutely Abnormal Number ■ Möbius Function ■ Redheffer Matrix ■ Möbius Inversion ■ Calculating Fourier Transforms through Möbius Inversion ■ Jacobi Symbol ■ Reciprocity Law

2.3 Combinatorial Functions

Factorials ■ Digits of Factorials ■ Stirling's Formula ■ Binomials and Multinomials ■ Nested Triangle Patterns ■ Stirling Numbers ■ Counting Partitions ■ Generating Partitions ■ Partition Identities

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▷ ▷ **Overview**

▷ ▷ **Exercises**

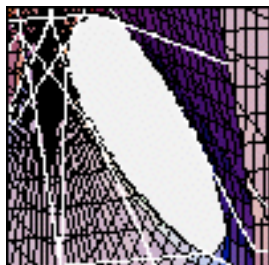
Sum of Divisor Powers ■ Recurrence Relation for Primes ■ Arcsin Law for Divisors ■ Average Length of Continued Fractions of Rationals ■ Isenkrahe Algorithm ■ Prime Divisors ■ Kimberling Sequence ■ Cantor Function Integral ■ Cattle Problem of Archimedes ■ Mirror Charges in a Wedge ■ Periodic Decimal Numbers ■ Digit Sequences in Numbers ■ Numbered Permutations ■ Binomial Coefficient Values ■ Smith's Sturmian Word Theorem ■ Modeling a Galton Board ■ Ehrenfest Urn Model ■ Ring Shift Modeling ■ Sandpile Model ■ Longest Common Subsequence ■ Riffle Shuffles ■ Weekday from Date ■ Easter Dates ■ Lattice Points in Disks ■ Binomial Digits ■ Average of Partitions ■ Partition Moments ■ 15 and 6174 ■ Selberg Identity ■ Kluver Identities ■ Ford Circles ■ Farey–Brocot Interval Coverings ■ Sum of Primes ■ Visualizing Eisenstein Series ■ Magnus Expansion ■ Rademacher Identity ■ Goldbach Conjecture ■ Zeckendorf Representation ■ Sylvester–Fibonacci Expansion ■ Ramanujan τ Function ■ Cross-Number Puzzle ■ Cyclotomic Polynomials ■ Generalized Bell Polynomials ■ Online Bin Packings ■ Composition Multiplicities ■ Subset Sums

▷ ▷ Solutions

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▷ ▷ References

S Y M B O L I C S



CHAPTER 1

Symbolic Computations

1.0 Remarks

1.1 Introduction

General Assumptions about Variables ■ Simplifying Expressions ■ Type Declarations for Simplifications ■ Evaluating Expressions Under Assumptions

1.2 Operations on Polynomials

1.2.0 Remarks

1.2.1 Structural Manipulations on Polynomials

Expanding and Factoring Polynomials ■ Factors of Random Polynomials ■ Irreducible Polynomials ■ Constructing Irreducible Polynomials from Primes ■ Factorization over Extension Fields ■ Reordering Multivariate Polynomials ■ Indeterminates of Polynomials ■ Extracting Coefficients from Polynomials ■ Decomposing Polynomials

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The Notion of Generic Solutions ■ Solving Univariate Polynomials in Radicals ■ Cubic Polynomials with Three Real Roots ■ Symbolic Roots as Solutions of Univariate Polynomials of Any Degree ■ Exact Operations on Polynomial Roots ■ Matrix Eigenvalues ■ Canonicalization of Symbol-Free Algebraic Expression ■ Hölder's Theorem about Real Roots of Cubics ■ Solving Systems of Polynomials ■ Vieta Relations ■ Solving Systems of Algebraic Equations ■ Solving Nonpolynomial Equations ■ Using Inverse Functions ■ Solving Trigonometric Equations ■ Solving Transcendental Equations ■ Verifying Parametric Solutions ■ Superposition of Damped Oscillations ■ Finding Degenerate Solutions ■ Elimination of Variables ■ Universal Differential Equation ■ Guidelines for Solving Equations and Systems of Equations

1.6 Classical Analysis

1.6.1 Differentiation

Multivariate Differentiation ■ Numericalization of Unevaluated Derivatives ■ Numerical Differentiation ■ Differentiating in the Complex Plane ■ Schwarz Theorem ■ Differential Algebraic Constants ■ High-Order Derivatives ■ Derivatives of Inverse Functions ■ Differentiation With Respect to Vectors ■ Derivatives of Pure Functions ■ Adding New Differentiation Rules ■ Differential Equations for n -Nomials ■ Generalized Taylor Expansion ■ Differentials ■ Metric Tensors, Christoffel Symbols, and Geodesics ■ Iterated Evolutes ■ Phase Integral Approximation

1.6.2 Integration

Algorithms for Symbolic Integration ■ Assumptions on Variables Having Generic Values ■ Integrating Abstract Functions ■ Korteweg–deVries Equation Hierarchy ■ Indefinite Integration Samples ■ Integrals and Special Functions ■ Integrating Rational Functions ■ Integrating Algebraic Functions ■ Assumptions of Parameter Variables ■ Assumptions in Indefinite Integrals ■ Generating Conditions for Convergence ■ Divergent and Hadamard-Regularized Integrals ■ Cauchy Principal Value Integrals ■ Multidimensional Integrals ■ Robbin's Integral Identity ■ Definite Integrals from Indefinite Integrals ■ Piecewise Continuous Antiderivatives ■ Continuity of Indefinite Integrals ■ Weierstrass Parametrization of Minimal Surfaces ■ Infinite Resistor Network ■ Timings of Indefinite versus Definite Integration ■ d'Alembert Solution of the One-Dimensional Wave Equation ■ Schrödinger Equation with a Time-dependent Linear Potential ■ Definite Integrals and Branch Cuts

1.6.3 Limits

Indeterminate Expressions and Limits ■ Limit Samples ■ Direction Dependence of Limits ■ Evaluating Limits Under Assumptions ■ Limits of Analytic Functions ■ Schwarz Derivative ■ Extracting Leading Terms ■ Limits of Iterative Function Applications ■ Multiple Limits

1.6.4 Series Expansions

Internal Structure of a Series-Object ■ Taylor Series ■ Continued Fraction with Three Limit Points ■ Laurent Series ■ Puiseux Series ■ Series Expansions at Branch Points and Branch Cuts ■ Series of Special Functions ■ Essential Singularities ■ Numerov–Mickens Scheme ■ Multivariate Series ■ Roots of Truncated Series ■ q -Taylor Series ■ Arithmetic of Series ■ Change for \$1 ■ Iterated Constant Terms ■ Inverse Series ■ Higher-Order Newton and Chebyshev Methods ■ Fractional Iterations ■ Cumulant Expansions ■ Laurent Series for Mandelbrot Set ■ Approximating Linear Functionals

1.6.5 Residues

Symbolic Residues at Poles ■ Generalized Residues ■ Residues of Special Functions

1.6.6 Sums

Sum of Powers ■ Numericalization of Symbolic Expressions ■ Procedural versus Symbolic Finite Summations ■ Riemann Surface of the Square Root Function ■ Weierstrass's Method of Analytic Continuation

1.7 Differential and Difference Equations

1.7.0 Remarks

1.7.1 Ordinary Differential Equations

Solutions as Rules ■ Pure Functions as Solutions ■ Degenerate Solutions ■ Differential Equation for Free Fall Including the Coriolis Force ■ Integration Constants ■ Linear Inhomogeneous ODE with Constant Coefficients ■ ODEs with Separated Variables ■ Homogeneous ODEs ■ Exact ODEs ■ Bernoulli ODE ■ Jacobi ODEs ■ Special Riccati ODEs ■ Abel ODEs of the First Kind ■ Abel ODEs of the Second Kind ■ Chini ODEs ■ Lagrange ODEs ■ Clairaut ODEs ■ ODEs with Shifted Argument ■ Cayley ODE ■ Second Order ODEs ■ Differential Equations of Special Functions ■ Schrödinger Equations for Various Smooth Potentials ■ Schrödinger Equations for Piecewise-Defined Potentials ■ Higher-Order Differential Equations ■ Implicit Solutions ■ Monitoring Differential Equation Solving ■ δ -Expansion

1.7.2 Partial Differential Equations

Hamilton–Jacobi Equation ■ Szebehely's Equation ■ Solutions with Arbitrary Functions

1.7.3 Difference Equations

Linear Difference Equations ■ Calculating Casoratians ■ Linear Difference Equations with Nonconstant Coefficients ■ Some Nonlinear Difference Equations ■ Difference Equations Corresponding to Differential Equations ■ Systems of Difference Equations

1.8 Integral Transforms and Generalized Functions

Generalized Functions and Linear Functionals ■ Heaviside Theta Function and Dirac Delta Function ■ Integrals Containing Generalized Functions ■ Multivariate Heaviside Theta and Dirac Delta Function ■ Time Dilation ■ Derivatives of the Dirac Delta Function ■ Simplifying Generalized Functions ■ Sequence Representations of Generalized Functions ■ Green's Function of Linear Differential Operators ■ Generalized Solutions of Differential Equations ■ Compactons ■ Fourier Transforms ■ Self-Fourier Transform ■ Principle Value Distribution ■ Sokhotsky–Plemelj Formula ■ Poincaré–Bertrand Identity ■ Laplace Transforms ■ Borel Summation of Divergent Sums ■ Adomian Decomposition

1.9 Additional Symbolic Functions

Variational Calculus ■ Symbolic Series Terms ■ Ramanujan's Master Theorem

1.10 Three Applications

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1.10.1 Area of a Random Triangle in a Square

A Quote from M. W. Crofton ■ Generalizations ■ Generic Cylindrical Algebraic Decompositions ■ Six-Dimensional Definite Integrals from Indefinite Integrals ■ Monte Carlo Modeling ■ Calculating the Probability Distribution of the Area

1.10.2 $\cos(\frac{2\pi}{257})$ à la Gauss

The Morning of March 29 in 1796 ■ Gauss Periods ■ Primitive Roots ■ Splitting and Combining Periods ■ Thousands of Square Roots ■ $\cos(\frac{2\pi}{65537})$ ■ Fermat Primes

1.10.3 Implicitization of a Trefoil Knot

Parametric versus Implicit Description of Surfaces ■ Envelope Surface of a Moving Ball ■ Polynomialization of Trigonometric Expressions ■ Calculating a Large Resultant ■ Smoothing the Trefoil Knot ■ Inflating a Trefoil Knot ■ Implicit Klein Bottle

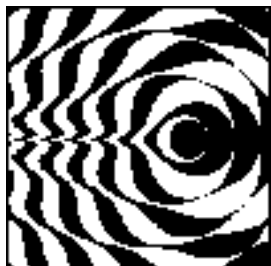
▷ ▷ **Overview**▷ ▷ **Exercises**

Heron's formula ■ Tetrahedron Volume ■ Apollonius Circles ■ Proving Trigonometric Identities ■ Icosahedron Inequalities ■ Two-Point Taylor Expansion ■ Horner Form ■ Nested Exponentials and Logarithms ■ Minimal Distance between Polynomial Roots ■ Dynamical Determinants ■ Appell–Nielsen Polynomials ■ Scoping in Iterated Integrals ■ Rational Solution of Painlevé II ■ Differential Equation for Products and Quotients of Linear Second Order ODEs ■ Singular Points of First-Order ODEs ■ Fredholm Integral Equation ■ Inverse Sturm–Liouville Problem ■ Graeffe Method ■ Lagrange Interpolation in 2D Triangles ■ Finite Element Matrices ■ Hermite Interpolation-Based Finite Element Calculations ■ Hylleraas–Undheim Helium Ground State Calculation ■ Variational Calculations ■ Hyperspherical Coordinates ■ Constant Negative Curvature Surfaces ■ Optimal Throw Angle ■ Jumping from a Swing ■ Normal Form of Sturm–Liouville Problems ■ Noncentral Collisions ■ Envelope of the Bernstein Polynomials ■ Eigensystem of the Bernstein Operator ■ A Sensitive Linear System ■ Bisector Surfaces ■ Smoothly Connecting Three Half-Infinite Cylinders ■ Nested Double Tori ■ Changing Variables in PDEs ■ Proving Matrix Identities ■ A Divergent Sum ■ Casimir Effect Limit ■ Generating Random Functions ■ Numerical Techniques Used in Symbolic Calculations ■ Series Solution of the Thomas–Fermi Equation ■ Majorana Form of the Thomas–Fermi Equation ■ Yoccoz Function ■ Lagrange–Bürmann Formula ■ Divisor Sum Identities ■ Eisenstein Series ■ Product Representation of \exp ■ Multiple Differentiation of Vector Functions ■ Expressing Trigonometric Values in Radicals ■ First Order Modular Transformations ■ Forced Damped Oscillations ■ Series for Euler's Constant ■ q -Logarithm ■ Symmetrized Determinant ■ High Order WKB Approximation ■ Greenberger–Horne–Zeilinger State ■ Entangled Four Particle State ■ Integrating Polynomial Roots ■ Riemann Surface of a Cubic ■ Series Solution of the Kepler Equation ■ Short Time-Series Solution of Newton's Equation ■ Lagrange Points of the Three-Body Problem ■ Implicitization of Lissajou Curves ■ Evolutes ■ Orthopodic Locus of Lissajou Curves ■ Cissoid of Lissajou Curves ■ Multiple Light Ray Reflections ■ Hedgehog Envelope ■ Supercircle Normal Superpositions ■ Discriminant Surface ■ Periodic Surface ■ 27 Lines on the Clebsch Surface ■ 28 Bitangents of a Plane Quartic ■ Pentaellipse ■ Galilean Invariance of Maxwell Equations ■ Relativistic Field Transformations ■ X-Waves ■ Thomas Precession ■ Liénard–Wiechert Potential Expansion ■ Spherical Standing Wave ■ Ramanujan's Factorial Expansion ■ q -Series to q -Products ■ q -Binomial ■ Multiplicative Series ■ gcd-Free Partitions ■ Single Differential Equations for Nonlinear Systems ■ Lattice Green's Function Differential Equation ■ Puzzles ■ Newton–Leibniz Theorem in 2D ■ Square Root of Differential Operator ■ Polynomials with Identical Coefficients and Roots ■ Amoebas ■ Cartesian Leaf Area ■ Average Distance between Random Points ■ Series Solution for Duffing Equation ■ Secular Terms ■ Implicitization of Various Surfaces ■ Kronig–Penney Model Riemann Surface ■ Ellipse Secants Envelope ■ Lines Intersecting Four Lines ■ Shortest Triangle Billiard Path ■ Weak Measurement Identity ■ Logarithmic Residue ■ Geometry Puzzle ■ Differential Equations of Bivariate Polynomials ■ Graph Eigenvalues ■ Change of Variables in the Dirac Delta Function ■ Probability Distributions for Sums ■ Random Determinants ■ Integral Representation of Divided Differences ■ Fourier Transform and Fourier Series ■ Functional Differentiation ■ Operator Splitting Formula ■ Tetrahedron of Maximal Volume

▷ ▷ **Solutions**

ODE for Circles ■ Modular Equations ■ Converting Trigonometric Expressions into Algebraic Expressions ■ Matrix Sign Function ■ Integration with Scoping ■ Collecting Powers and Logarithms ■ Bound State in Continuum ■ Element Vectors, Mass Matrices, and Stiffness Matrices ■ Multivariate Minimization ■ Envelopes of Throw Trajectories ■ Helpful Warning Messages ■ Using Ansätze ■ Schanuel's Conjecture ■ Matrix Derivatives ■ Lewis–Carroll Identities ■ Abel and Hölder Summation ■ Extended Poisson Summation Formula ■ Integration Testing ■ Detecting the Hidden Use of Approximate Numbers ■ Functions with Nontrivial Derivatives ■ Expressing ODEs as Integral Equations ■ Finding Modular Null Spaces ■ Canonicalizing Tensor Expressions ■ Nonsorting “Unioning” ■ Linear Diophantine Equations ■ Ramanujan Trigonometric Identities ■ Cot Identities ■ Solving the Fokker–Planck Equation for the Forced Damped Oscillator ■ Implementing Specialized Integrations ■ Bras and Kets ■ Density Matrices ■ Recognizing Algebraic Numbers ■ Differentiation of Symbolic Vectors ■ Visualizing the Lagrange Points ■ Gröbner Walk ■ Piecewise Parametrizations of Implicit Surfaces ■ Generalized Clebsch Surfaces ■ Algorithmic Rewriting of Covariant Equations in 3D Vectors ■ Darboux–Halphen System ■ Cubed Sphere Equation ■ Numerically Checking Integrals Containing Derivatives of Dirac Delta Functions ■ Lagrange Multipliers ■ Elementary Symmetric Polynomials

▷ ▷ **References**



CHAPTER 2

Classical Orthogonal Polynomials

2.0 Remarks

2.1 General Properties of Orthogonal Polynomials

Orthogonal Polynomials as Solutions of Sturm–Liouville Eigenvalue Problems ■ General Properties of Orthogonal Polynomials ■ Expansion of Arbitrary Functions in Orthogonal Polynomials

2.2 Hermite Polynomials

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Harmonic Oscillator Eigenfunctions ■ Density of States ■ Shifted Harmonic Oscillator

2.3 Jacobi Polynomials

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Electrostatic Interpretation of the Zeros ■ Pöschl–Teller Potential

2.4 Gegenbauer Polynomials

Laplace Equation in nD ■ Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Smoothing the Gibbs Phenomenon

2.5 Laguerre Polynomials

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Expanding Riemann Spheres ■ Summed Atomic Orbitals

2.6 Legendre Polynomials

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Associated Legendre Polynomials ■ Modified Pöschl–Teller Potential

2.7 Chebyshev Polynomials of the First Kind

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Trigonometric Form ■ Special Properties

2.8 Chebyshev Polynomials of the Second Kind

Definition ■ Graphs ■ ODE ■ Orthogonality and Normalization ■ Trigonometric Form

2.9 Relationships Among the Orthogonal Polynomials

Gegenbauer Polynomials as Special Cases of Jacobi Polynomials ■ Hermite Polynomials as Special Cases of Associated Laguerre Polynomials ■ Relations between the Chebyshev Polynomials ■ Calogero–Sutherland Model ■ Schmeisser Companion Matrix ■ Iterated Roots of Orthogonal Polynomials

2.10 Ground-State of the Quartic Oscillator

Harmonic and Anharmonic Oscillators ■ Matrix Elements in the Harmonic Oscillator Basis ■ High-Precision Eigenvalues from Diagonalizing the Hill Matrix ■ Lagrange Interpolation-Based Diagonalization ■ Complex Energy Surfaces ■ Time-Dependent Schrödinger Equation ■ \mathcal{PT} -Invariant Oscillators

▷ ▷ Overview

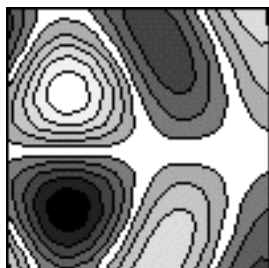
▷ ▷ Exercises

Mehler's Formula ■ Addition Theorem for Hermite Polynomials ■ Sums of Zeros of Hermite Polynomials ■ Spherical Harmonics ■ Sums of Zeros ■ General Orthogonal Polynomials ■ Gram-Schmidt Orthogonalization ■ Power Sums ■ Elementary Symmetric Polynomials ■ Newton Relations ■ Waring Formula ■ Generalized Lissajous Figures ■ Hyperspherical Harmonics ■ Hydrogen Orbitals ■ Zeros of Hermite Functions for Varying Order ■ Ground State Energy of Relativistic Pseudodifferential Operator ■ Moments of Hermite Polynomial Zeros ■ Coherent States ■ Smoothed Harmonic Oscillator States ■ Darboux Isospectral Transformation ■ Forming Wave Packets from Superpositions ■ Multidimensional Harmonic Oscillator ■ High-Order Perturbation Theory ■ Differential Equation System for Eigenvalues ■ Time-Dependent Sextic Oscillator ■ Time Dependent Schrödinger Equation with Calogera Potential

▷ ▷ Solutions

Bauer–Rayleigh Identity ■ Parseval Identity ■ Transmission through Periodic Structures ■ Freud's Weight Function ■ Wronski Polynomials ■ Root-Finding Using Differential Equations ■ Finding Ramification Indices Numerically ■ Classical and Quantum Mechanical Probabilities for the Harmonic Oscillator ■ Root Approximant ■ Using Recursion Relations to Calculate Orthogonal Polynomials

▷ ▷ **References**



CHAPTER 3

Classical Special Functions

3.0 Remarks

Information Sources about Special Functions ■ Experimental Mathematics ■ Generalized Harmonic Numbers ■ Position and Momentum Eigenfunctions and Wigner Function of the Liouville Potential ■ Ramanujan Theta Functions ■ Modular Identities

3.1 Introduction

Simplifying Expressions Containing Special Functions ■ Expressing Special Functions through Simpler Ones ■ Indefinite Integrals of Compositions of Elementary Functions ■ Volume of a Supersphere ■ PT -Symmetric Oscillator ■ Monitoring Simplifying Transformations

3.2 Gamma, Beta, and Polygamma Functions

Definitions ■ Exact Values ■ Graphs ■ Riemann Surface of the Incomplete Gamma Function ■ Pochhammer Symbol

3.3 Error Functions and Fresnel Integrals

Definitions ■ Error Function in the Complex Plane ■ Iterated Integrals of Error Functions ■ Free Particle Schrödinger Equation with Piecewise Constant Initial Conditions ■ Moshinsky Function ■ Harmonic Oscillator Green's Function ■ Fresnel Diffraction on a Half-Plane

3.4 Exponential Integral and Related Functions

Definitions ■ Graphs ■ Logarithmic Integral and Prime Counting Function

3.5 Bessel and Airy Functions

Definitions ■ Random Walk on a 2D Square Lattice ■ Fractal Based on Bessel Function ■ Weber–Schafheitlin Integrals ■ Bessel Zeros as a Function of the Index ■ Oscillation of a Circular Drum ■ Oscillation of a Drum of General Shape ■ 2D Helmholtz Equation ■ Eigenvalues and Eigenfunctions of the Stadium Billiard ■ Free Nonspreading Wave “Packet” ■ Airy Functions in the Uniform Approximation of Linear Turning Point Problem ■ Harmonic Oscillator Approximations

3.6 Legendre Functions

Definitions ■ Graphs ■ Electrostatic Potential in a Conducting Cone

3.7 Hypergeometric Functions

Gauss Hypergeometric Function and Generalized Hypergeometric Functions ■ Some Special Cases ■ Closed Form of Partial Sums of Taylor Series for Trigonometric Functions ■ Closed Form Padé Approximations of \exp and sign ■ Generalized Fresnel Integrals ■ Generalized Exponential Functions ■ Point Charge Outside a Dielectric Sphere ■ Finding Contiguous Relations ■ Regularized Hypergeometric Functions ■ Solutions of the Hypergeometric Differential Equation ■ Meijer G Function ■ Eigenfunctions of the Inverse Harmonic Oscillator ■ Bivariate Hypergeometric Functions

3.8 Elliptic Integrals

Integrals Containing Square Roots of Cubics and Quartics ■ Definitions ■ Complete and Incomplete Elliptic Integrals ■ Graphs ■ Deriving Differential Equations for Incomplete Elliptic Integrals ■ Green's Function of the Zeilon Operator ■ Finding Modular Equations for Ratios of Elliptic Integrals

3.9 Elliptic Functions

Inverting Elliptic Integrals ■ Definitions ■ Jacobi's Amplitude Function ■ Minimal Surface in a Cube Wireframe ■ Applications of Elliptic Functions ■ Pendulum Oscillations ■ Current Flow through a Rectangular Conducting Plate ■ Arithmetic–Geometric Mean

3.10 Product Log Function

Definition ■ Solving Transcendental Equations ■ Riemann Surface of the Product Log Function

3.11 Mathieu Functions

Differential Equation with Periodic Coefficients ■ Definition ■ Characteristic Values ■ Resonance Tongues ■ Branch Cuts and Branch Points ■ Oscillation of an Ellipsoidal Drum ■ Degenerate Eigenfunctions ■ Wannier Functions

3.12 Additional Special Functions

Expressing Other Special Functions through Built-in Special Functions ■ More Elliptic Functions ■ Zeta Functions and Lerch Transcendents

3.13 Solution of Quintic Polynomials

Solving Polynomials in Radicals ■ Klein's Solution of the Quintic ■ Tschirnhaus Transformation ■ Principal Quintic ■ Belyi Function and Stereographic Projection of an Icosahedron Projection ■ Solving a Polynomial of Degree 60 through Hypergeometric Functions ■ Numerical Root Calculation Based on Klein's Formula

▷ ▷ **Overview**

▷ ▷ **Exercises**

Asymptotic Expansions of Bessel Functions ■ Carlitz Expansion ■ Meissel's Formula ■ Rayleigh Sums ■ Gumbel Distribution ■ Generalized Bell Numbers ■ Borel Summation ■ Bound State in Continuum ■ ODEs for Incomplete Elliptic Integrals ■ Addition Formulas for Elliptic Integrals ■ Magnetic Field of a Helmholtz Coil ■ Identities, Expansions, ODEs, and Visualizations of the Weierstrass \wp Function ■ Sutherland–Calogero Model ■ Weierstrass Zeta and Sigma Functions ■ Lamé Equation ■ Vortex Lattices ■ ODEs, Addition Formulas, Series Expansions for the Twelve Jacobi Elliptic Functions ■ Schrödinger Equations with Potentials that are Rational Functions of the Wave Functions ■ Periodic Solutions of Nonlinear Evolution Equations ■ Complex Pendulum ■ Harmonic Oscillator Eigenvalues ■ Contour Integral Representation of Bessel Functions ■ Large Order and Argument Expansion for Bessel Functions ■ Aperture Diffraction ■ Circular Andreev Billiard ■ Contour Integral Representation for Beta Functions ■ Beta Distribution ■ Euler's Constant Limit ■ Time-Evolution in a Triple-Well Oscillator ■ Eigenvalues of a Singular Potential ■ Dependencies in the Numerical Calculation of Special Functions ■ Hidden Derivative Definitions ■ Perturbation Theory for a Square Well in an Electric Field ■ Oscillations of a Pendulum with Finite Mass Cord ■ Approximation and Asymptotics of Fermi–Dirac Integrals ■ Sum of All 9-Free Reciprocal Numbers ■ Green's Function for 1D Heat Equation ■ Green's Function for the Laplace Equation in a Rectangle ■ Addition Theorems for Theta Functions ■ Series Expansion of Theta Functions ■ Bose Gas in a 3D Box ■ Scattering on a Conducting Cylinder ■ Poincaré Waves ■ Scattering on a Dielectric Cylinder ■ Coulomb Scattering ■ Spiral Waves ■ Scattering on a Corrugated Wall ■ Random Helmholtz Equation Solutions ■ Toroidal Coordinates ■ Riemann–Siegel Expansion ■ Zeros of the Hurwitz Zeta Function ■ Zeta Zeta Function ■ Harmonic Polylogarithms ■ Riemann Surface of Gauss Hypergeometric Functions ■ Riemann Surface of the Ratio of Complete Elliptic Integrals ■ Riemann Surface of the Inverse Error Function ■ Kummer's 24 Solutions of Gauss Hypergeometric Equation ■ Differential Equation for Appell Function ■ Gauss–Lucas Theorem ■ Roots of Differentiated Polynomials ■ Coinciding Bessel Zeros ■ Ramanujan π Formulas ■ Force-Free Magnetic Fields ■ Bessel Beams ■ Gauge Transformation for a Square ■ Riemann Surface of the Bootstrap Equation ■ Differential Equations for Powers of Airy Functions ■ Asymptotic Expansions for the Zeros of Airy Functions ■ Map–Airy Distribution ■ Dedekind η ODE ■ Darboux–Halphen System ■ Ramanujan Identities for φ and λ Functions ■ Generating Identities in Gamma Functions ■ Modular Equations for Dedekind η Function

▷ ▷ **Solutions**

Truncation of Asymptotic Series ■ Contour Plots of the Gamma Function ■ Series of a Gamma Function Ratio ■ Partial Sums of Taylor Series for \sin ■ Area and Volume of a Hypersphere ■ All Integrals of Three Compositions of Elementary Functions ■ Binomial at Negative Integers ■ Contour Lines of z^z ■ Weierstrass \wp Function over the Riemann Sphere ■ Using Gröbner Bases to Derive ODEs ■ Riemann Surface of Inverse Weierstrass \wp Function ■ Rocket with Discrete Propulsion ■ Monitoring All Internal Calculations ■ Machine versus High-Precision Evaluations of Special Functions ■ Checking Numerical Function Evaluation ■ Zeta Regularized Divergent Products ■ Fractional Derivatives ■ Identifying Algebraic Numbers

▷ ▷ References

A P P E N D I C E S



APPENDIX A

General References to Computer Algebra and to Mathematica**A.0 Remarks****A.1 References and Other Sources of Information**

- A.1.1 General References on Algorithms for Computer Algebra
 - General Computer Algebra Books, References, and Websites ■ Sources of Algorithms ■ Computer Algebra Journals and Conferences
- A.1.2 Comparison of Various Systems
 - Benchmarks and Timing Comparisons
- A.1.3 References on *Mathematica*
 - Books ■ Journals and Websites ■ Conferences ■ Package Libraries ■ Dedicated Newsgroups ■ Timing Comparisons
- A.1.4 Applications of Computer Algebra Systems
 - Article Samples ■ Further Information Sources

▷ ▷ **References**



APPENDIX B (from <http://www.mathematicaguidebooks.org>)

The Front End, the Help Browser, Notebooks, Stylesheets, Cells, Typesetting, Buttons, Boxes, and All That

B.0 Remarks

B.1 Notebooks and Cells as Expressions

- B.1.1 The Structure of Notebook
- B.1.2 The Appearance of Cells
- B.1.3 Stylesheets
- B.1.4 Selected Cell Options

B.2 Front End Functions and Operations

- B.2.1 Navigating and Manipulating Notebook
- B.2.2 Performing Menu and Keyboard Operations Programmatically

B.3 Typesetting and Boxes

- B.3.1 Two-Dimensional Formatting
- B.3.2 Tweaking Formula Appearances
- B.3.3 Creating Typesetting Rules

B.4 Buttons, Hyperlinks, and Palettes

- B.4.1 General Buttons
- B.4.2 Hyperlinks
- B.4.3 Palettes

B.5 Dynamic Boxes

- B.5.1 Automatic Numbering
- B.5.2 Displaying Values Automatically

B.6 Special Notebooks

- B.6.1 Help Browser Notebooks
- B.6.2 The Message Notebook

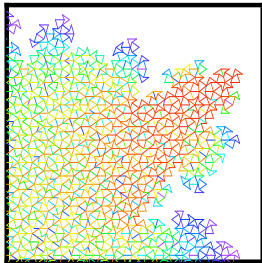
B.7 MathLink-Related Operations

B.8 Three Applications

- B.8.0 Remarks
- B.8.1 Analyzing the Notebook Version of *The Mathematica Book*
- B.8.2 Incorporating the *GuideBooks* into the Help Browser
- B.8.3 Evaluating a Complete *GuideBooks* Chapter Programmatically

▷ ▷ **References**

A D D I T I O N S



ADDITIONS FROM THE WEBSITE <http://www.mathematicaguidebooks.org>

Additional Exercises and Solutions

W.0 Remarks

W.7 Additions to Chapter 1 of the Graphics Volume

Repeated Breaking of a Stick ■ Animation of Rotating Tiles of an Aperiodic Tiling ■ Animation of Circles on Lissajou Figures

W.8 Additions to Chapter 2 of the Graphics Volume

Animation of Rotating Textured Möbius Bands ■ Animation Of Rotating Interlocked Tori ■ Klein Bottle with Hexagonal Massive Wireframe ■ Many Random Walkers in 3D ■ Bivariate Minkowski Function ■ Farey and Bary Addition ■ Projections from 4D

W.9 Additions to Chapter 3 of the Graphics Volume

Animation of Equal-Eigenvalue Chladny Figures ■ Animation of Moving Charged Regular Polygons ■ Graphics of Charged Truchet Patterns

W.10 Additions to Chapter 1 of the Numerics Volume

Random Walks with Variable Stepsize ■ Chaotic Scattering on Three Disks ■ Vibrating 2D Hilbert Curve ■ Optimal Projections of Hypercubes ■ Currents Through a Penrose Tiling ■ Numerical Solutions of Various Partial Differential Equations ■ Brain Growth Modeling ■ Step Bunching Modeling ■ Swift–Hohenberg Equation ■ Meinhardt Equations ■ Complex Ginzburg–Landau Equation Hierarchy ■ Splitting Localized Structures ■ Wave Equation with Piecewise-Constant Phase Velocities ■ Local Induction Approximation ■ Born–Infeld Wave Equation ■ Peakon Trains ■ Vibrations of a Square Koch Drum ■ Weyl–Berry Law ■ Diverging Gradients at Inner Corners ■ Classical and Quantum Mechanical Treatment of a Duffing Oscillator ■ Calculating Wigner Functions Through Fractional Fourier Transforms ■ Sub- \hbar Structures in the Wigner Function ■ Circular Aperture Diffraction Integral ■ Checking the Cauchy–Born Hypothesis ■ Schwarz–Christoffel Map for Some Symmetric Polygons ■ Normalized Banzhaf Indices for the European Union Countries ■ Wave Propagation on a Torus Surface

W.11 Additions to Chapter 2 of the Numerics Volume

A Special Infinite Product of Cosines ■ A Special π -Related Continued Fraction ■ Plots of the Argument of Cyclotomic Polynomials

W.12 Additions to Chapter 1 of the Symbolics Volume

Convergence Radius of the Virial Series for the Bose Gas ■ Midpoint Quadrature Formula ■ MacMahon Master Theorem ■ Adler–Moser Polynomials ■ Differential Equation for Yablonskii–Vorob’ev Polynomials ■ Implicit Polynomial Description of A Hypocycloidal Torus ■ Calculating the Second Feigenbaum Constant ■ Green’s Function for a Sequence of Delta Function Potentials ■ Implicit Form of Poynting Vector Equisurfaces ■ Symmetrically Arranged Points on Spheres ■ The Isospectral Polygons Bilby and Hawk ■ Probability Distribution of a Quotient ■ Vibrations of Springs on a Gosper Curve ■ Probability Distribution for the Distance Between Two Points from the Unit Square ■ Animation of the Nodal Lines of a Dirichlet–Neumann Boundary Conditions Transition ■ Checking Higher Order Generalized WKB Approximation for the Harmonic Oscillator ■ Evaluating an Iterated Integral ■ The Kobussen–Leubner–Lopez Lagrangian for the Harmonic Oscillator

W.13 Additions to Chapter 2 of the Symbolics Volume

Eigenfunctions of the Hénon–Heiles Potential

W.14 Additions to Chapter 3 of the Symbolics Volume

Rational Values of the Gauss Hypergeometric Function ■ Eigenfunctions of the Discrete Harmonic Oscillator ■ Average Length of Smallest Component of Multidimensional Unit Vectors ■ Differential Equation of the Jacobi Elliptic Function sn with Respect to the Modul ■ A Certain Sum of Zeta Functions ■ High-Order Series Expansion of Harmonic Numbers of a Given Size ■ Movement of a Sliding Spinning Disk ■ Identities of Jacobi Theta Function for Special Argument Values

▷ ▷ References